Grade Level/Course: Grades 4 & 5

Lesson/Unit Plan Name: Decomposing Fractions –
Decompose Fractions as a sum and product of unit fractions using multiple methods

Rationale/Lesson Abstract: Having students introduced to multiplication as a method of decomposition of a fraction allows students to become familiar with this concept before moving to more complex problems. The use of decomposition and visual representations helps students reason about the different forms that fractions can be presented; improper fractions and mixed numbers. In this lesson students will decompose fractions using bar models, area models, and prime factorization to check their solutions.

Timeframe: 1 class session; 60 minutes

Common Core Standard(s): Number and Operations – Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

Grade 5: **5.NF.1, 5.NF.2, 5.NF.4, 5.NF.6, 5.NF.7**

Instructional Resources/Materials:

- White Boards
- Class Set of Fraction Tiles
- Crayons, Colored Pencils, and/or Highlighters
- Multiple Methods Recording Sheet or Plain copy paper folded into 4 x 2

Instructional Methods:

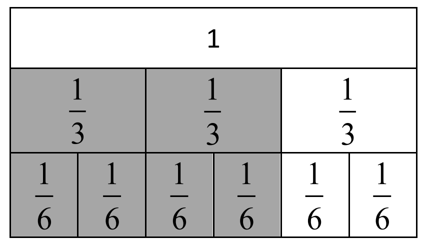
- Fraction Tiles – build a concrete model
- Bar Model – use the concrete model to help access the bar model method
- Area Model – use the bar mode to help access the area model method
- Addition Sentence – use the Bar Model to help write out the sum of the unit fractions
- Multiplication Sentence – use the Area Model to help write out the products of the unit fractions
- Prime Factorization – use prior knowledge of prime factorization to determine if fractions are equivalent

Review: Begin with warm-ups: Quadrant I, II, III, and IV

Lesson: Use multiple methods to demonstrate $\frac{2}{3}=\frac{4}{6}$.

Example 1: Bar Model

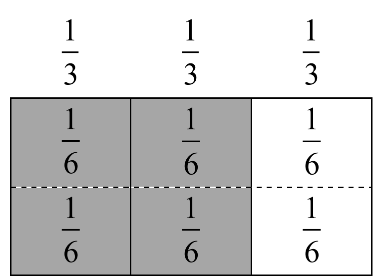
T: Place your 1 whole fraction tile on your desk, say the fraction.
S: One whole
T: Build 2 thirds, aligning the fraction tiles on the left.
T: Is $\frac{2}{3}$ less than or greater than one? Discuss with a partner.
S: Less than
T: Using the unit fraction sixths, find how many sixths are equivalent to $\frac{2}{3}$? Defend your solution to at least two of your classmates.
S: 4 sixths
T: Draw your bar model onto your recording sheet using the fraction tiles as a reference.
S: (See drawing)
T: Write a Therefore statement
S: $\therefore \frac{2}{3}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}$



$$\therefore \frac{2}{3}=\left(\frac{1}{6}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{6}\right)=\frac{4}{6}$$

Example 1: Area Model

T: Use the area model to show $\frac{2}{3}=\frac{4}{6}$.
S: Partition the area model into 3 equal parts and shade in 2 parts
T: Discuss with a partner how you would use the area model to decompose the thirds into sixths.
S: Draw a line horizontally dividing the three equal parts in to six equal parts.
T: How many sixths are shaded?
S: 4 sixths are shaded
T: Write a Therefore statement
S: $\therefore \frac{2}{3}=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)=\frac{4}{6}$



$$\therefore \frac{2}{3}=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)=\frac{4}{6}$$

Example 1: Mathematical Sentence using Sum of Unit Fractions

T: Write an addition sentence to illustrate the decomposition of $\frac{2}{3}=\frac{4}{6}$.

S: The shaded area stays the same size. The number of equal parts increase while the size of the equal parts decreases. Therefore 2 thirds is equivalent to 4 sixths.

$$\begin{aligned}\frac{2}{3}&=\frac{1}{3}+\frac{1}{3}\\&=\left(\frac{1}{6}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{6}\right)\\&=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)\\&=\frac{4}{6}\end{aligned}$$

$$\therefore\frac{2}{3}=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)=\frac{4}{6}$$

Example 1: Mathematical Sentence using Product of Unit Fractions

T: Write a multiplication sentence to illustrate the decomposition of $\frac{2}{3}=\frac{4}{6}$.

S: The shaded area stays the same size. The number of equal parts increase while the size of the equal parts decreases. Therefore if the unit fraction sixth is multiplied by $4\bullet\frac{1}{6}$ it still equals the area of the larger shaded area and is easier to write out than repeated addition.

$$\begin{aligned}\frac{2}{3}&=\frac{1}{3}+\frac{1}{3}\\&=\left(2\bullet\frac{1}{6}\right)+\left(2\bullet\frac{1}{6}\right)\\&=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)\\&=\frac{4}{6}\end{aligned}$$

$$\therefore\frac{2}{3}=4\bullet\frac{1}{6}=\frac{4}{6}$$

Example 1: Check Solution using Prime Factorization

T: Write $\frac{4}{6}$ and find the prime factors of the numerator and denominator. Simplify.

S: Prime factors of 4 and 6 are $2\bullet2$ and $2\bullet3$. Then find the equivalent form of one, $\frac{2}{2}$.

$$\frac{4}{6}=\frac{2\bullet2}{2\bullet3}=\frac{2}{3}$$

You Try #1:

Use multiple methods to show $\frac{2}{3}=\frac{}{9}$.

You Try #2:

Use multiple methods to show $\frac{2}{5}=\frac{}{10}$.

You Try #3:

Use multiple methods to show $\frac{3}{4}=\frac{}{8}$.

You Try #4:

Use multiple methods to show $\frac{3}{4}=\frac{}{12}$.

You Try #5:

Use multiple methods to show $\frac{4}{5}=\frac{}{10}$.

Debrief Questions:

- How can $\frac{2}{3}$ be equivalent to $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$?
- How can two different fractions represent the same part of a whole?
- What is the purpose of using parenthesis?
- What method do you prefer and why?

Decomposing Fractions using Multiple Methods to show Equivalency

Bar Model

Use a bar model to show that $\frac{2}{3}=\frac{}{6}$.

1					
$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\therefore \frac{2}{3}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}$$

Area Model

Use an area model to show that $\frac{2}{3}=\frac{}{6}$.

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\therefore \frac{2}{3}=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)=\frac{4}{6}$$

Mathematical Sentence

Sum of Unit Fraction

$$\frac{2}{3}=\frac{1}{3}+\frac{1}{3}$$
$$=\left(\frac{1}{6}+\frac{1}{6}\right)+\left(\frac{1}{6}+\frac{1}{6}\right)$$
$$=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)$$
$$=\frac{4}{6}$$

$$\therefore \frac{2}{3}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}$$

Product of Unit Fraction

$$\frac{2}{3}=\frac{1}{3}+\frac{1}{3}$$
$$=\left(2\bullet\frac{1}{6}\right)+\left(2\bullet\frac{1}{6}\right)$$
$$=\left(\frac{2}{6}\right)+\left(\frac{2}{6}\right)$$
$$=\frac{4}{6}$$

$$\therefore \frac{2}{3}=\left(4\bullet\frac{1}{6}\right)=\frac{4}{6}$$

Check your solution using prime factorization.

$$\frac{4}{6}=\frac{2\bullet 2}{2\bullet 3}=\frac{2}{3}$$

Date ____________________

Warm-Up

CCSS: 4.NF.3

Brenda is making cookies. The recipe calls for $\frac{2}{3}$ cup of chocolate chips. She only has a $\frac{1}{3}$ cup measuring cup. If she triples the recipe, how many times will she need to fill the $\frac{1}{3}$ cup with chocolate chips? Draw a bar model and record your solution as the product of the unit fraction.

Review: 3.NF.2

Compare the fractions below using at least two different ways.

$$\frac{2}{3} \bigcirc \frac{4}{10}$$

Current: 4.NF.4a

Given the expression, indicate which of the following are true or false.

$$3 \bullet \frac{3}{4}$$

a. $x = \left(\frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right)$ ① True ② False

b. $x = \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$ ① True ② False

c. $x = \frac{3}{4} \bullet \frac{3}{3}$ ① True ② False

d. $x = \frac{3}{4} \bullet \frac{3}{1}$ ① True ② False

e. $x = \frac{9}{4}$ ① True ② False

f. $x = \left(\frac{4}{4} + \frac{4}{4} + \frac{1}{4}\right)$ ① True ② False

g. $x = 2\frac{1}{4}$ ① True ② False

Other: 5.OA.2.1

Using prime factorization, what is $\frac{12}{60}$ expressed in lowest terms?

Format: PDG Jan. 2005

Answer Key

Warm-Up

CCSS: 4.NF.3	Review: 3.NF.2

Brenda is making cookies. The recipe calls for $\frac{2}{3}$ cup of chocolate chips. She only has a $\frac{1}{3}$ cup measuring cup. If she triples the recipe, how many times will she need to fill the $\frac{1}{3}$ cup with chocolate chips?

Draw a bar model and record your solution as the product of the unit fraction.

Original recipe			Doubled recipe			Tripled recipe		
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$$6 \bullet \frac{1}{3} = \frac{6}{3}$$

$\therefore$ Brenda will need to fill the $\frac{1}{3}$ measuring cup 6 times.

Compare the fractions below using at least two different ways.

$$\frac{2}{3} > \frac{4}{10}$$

$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{2}{3}$ is greater than half, $\frac{4}{10}$ is less than half

$\therefore$ 2 thirds is greater than 4 tenths

Current: 4.NF.4a	Other: 5.OA.2.1

Given the expression, indicate which of the following are true or false.

$$3 \bullet \frac{3}{4}$$

a. $x = \left(\frac{3}{4} + \frac{3}{4} + \frac{3}{4}\right)$ ❶ True ② False

b. $x = \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$ ① True ❷ False

c. $x = \frac{3}{4} \bullet \frac{3}{3}$ ① True ❷ False

d. $x = \frac{3}{4} \bullet \frac{3}{1}$ ❶ True ② False

e. $x = \frac{9}{4}$ ❶ True ② False

f. $x = \left(\frac{4}{4} + \frac{4}{4} + \frac{1}{4}\right)$ ❶ True ② False

g. $x = 2\frac{1}{4}$ ❶ True ② False

Using prime factorization, what is $\frac{12}{60}$ expressed in lowest terms?

$$\frac{12}{60} = \frac{2 \bullet 2 \bullet 3}{2 \bullet 3 \bullet 2 \bullet 5}$$

$$= \frac{2 \bullet 2 \bullet 3}{2 \bullet 2 \bullet 3 \bullet 5}$$

$$= \frac{1 \bullet 1 \bullet 1}{1 \bullet 1 \bullet 1 \bullet 5}$$

$$= \frac{1}{5}$$

$$\therefore \frac{12}{60} = \frac{1}{5}$$